# Fighting Coherent Noise With Pauli Gates

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## I. PROBLEM

**Coherent Errors**: unitary errors, usually due to errors in quantum control, e.g. systematic over-/under-rotation.

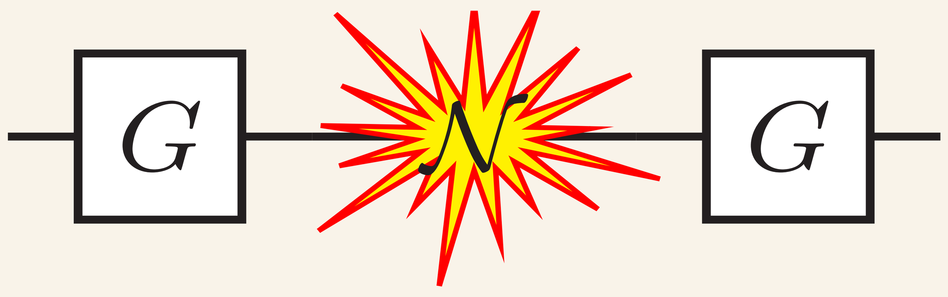
**Incoherent Errors**: probabilistic errors, usually due to interactions with the environment, e.g. dephasing channels.

For the same average error rate, the worst-case error rate of *coherent errors* can be much higher than the corresponding *incoherent error* [1].

## II. SOLUTION

Transforming coherent errors into incoherent errors via *Pauli twirling*.

**Pauli Twirling**: conjugating the error channel with a pair of *random* Pauli gates [2].




If conjugating the noise with random Pauli gates can achieve logical fidelity improvements, then deterministically conjugating the noise with an given optimal Pauli gate should performs even better than such random averaging.

**Pauli Conjugation**: *determintically* conjugating the error channel with a chosen Pauli gate.

## III. METHOD

Instead of testing all Pauli gates to find the optimal conjugation gate, we will only test one Pauli gate in each *equivalent class*.

**Equivalence Class**: a subset of gates within the Pauli gates that have the *same conjugation performance* when applied to a given noise process for a given quantum error correction code.

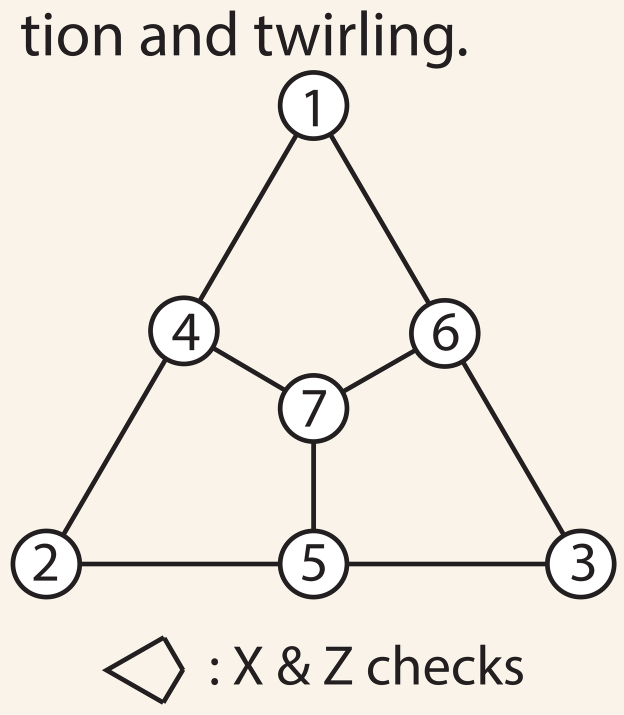
The Pauli gates are partitioned into different equivalent classes based on:

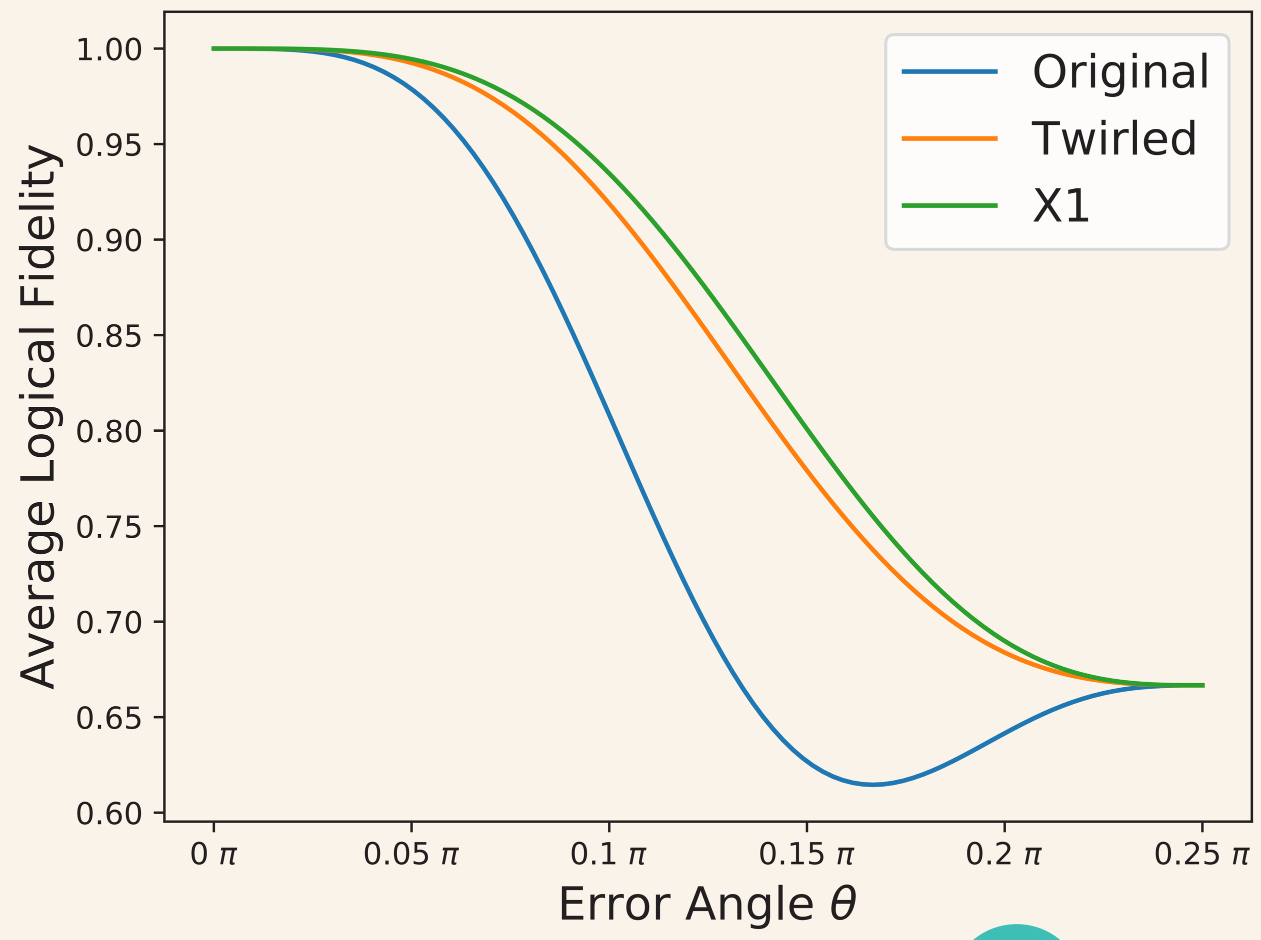
- The structure of the quantum error correction code
- The structure of the noise process
- The shared symmetries between the code and the noise

## IV. RESULT

**Example Noise Model**: global Z rotation $\exp(i\theta\sum_i Z_i)$

**Steane Code**: all Pauli gates are divided into two equivalence classes for conjugation. One is equivalent to I, the other is equivalent to $X_1$ (or any single-qubit flip). Here we can see that $X_1$ conjugation outperform both no conjugation and twirling.



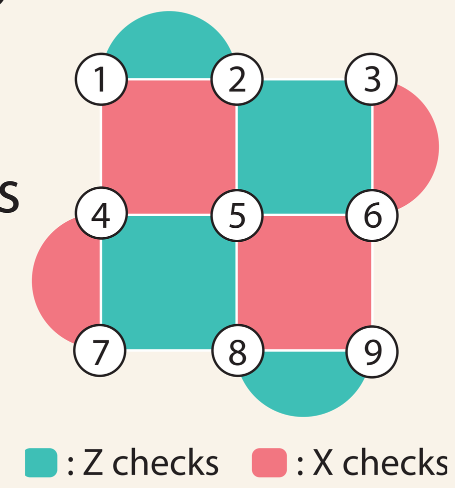



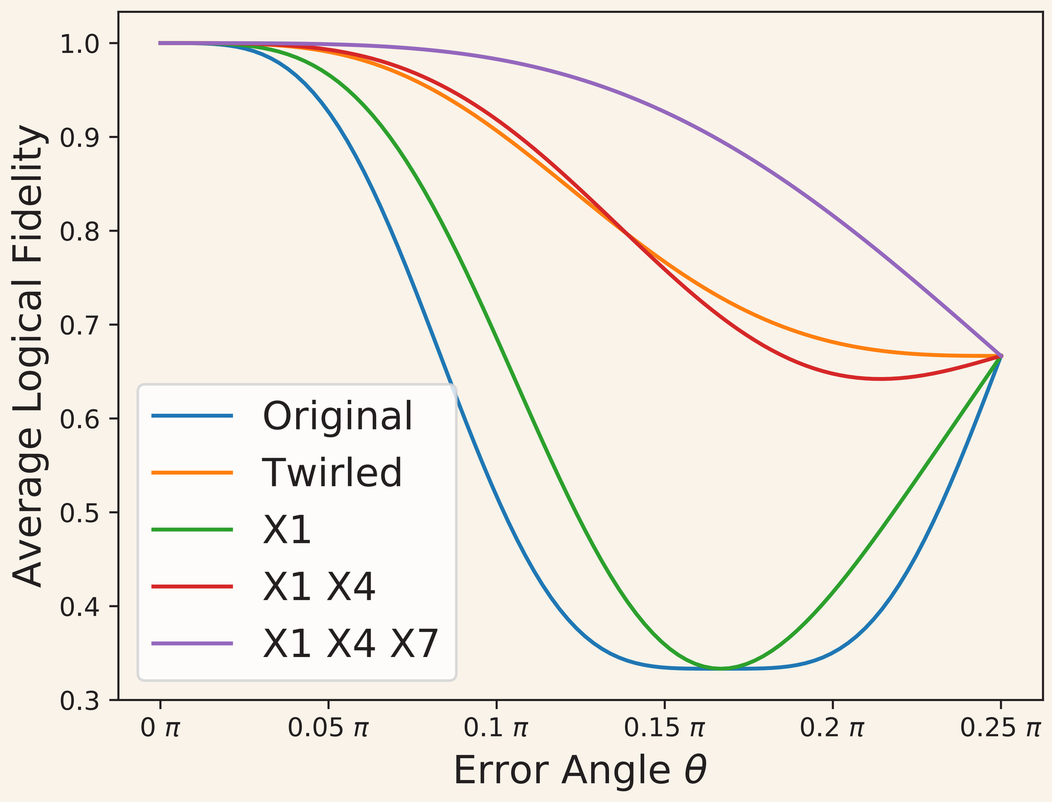

**Shor Code**:

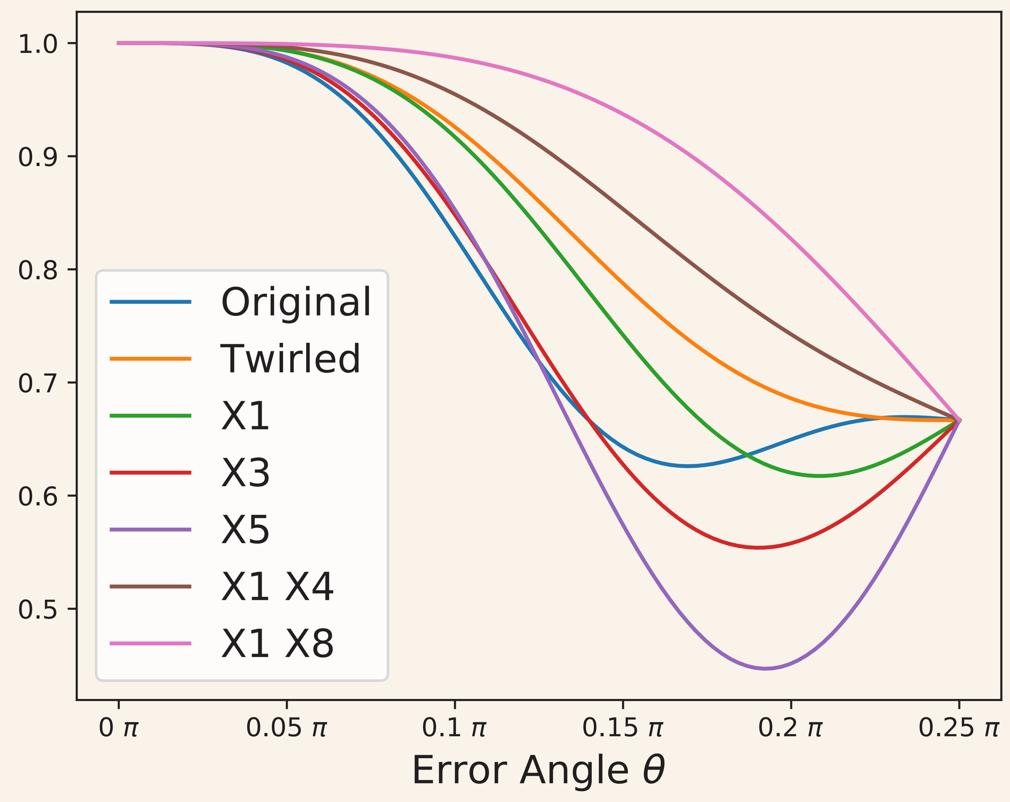
- 4 equivalent classes
- optimal conjugation gate: $X_1X_4X_7$


**Surface Code**:

- 6 equivalent classes
- optimal conjugation gate: $X_1X_8$




## V. CONCLUSION

**The advantages of Pauli conjugation:**

- **Cheap**: Pauli gates are usually easy to apply with low error rate.
- **Deterministic:** can be applied to hardware for which changing the circuit in each run is hard.
- **Logical Fidelity Improvement** over no cojugation and twirling as shown by our plots here.

**Challenges in finding the optimal conjugation gate** when we have

- Code and noise with little symmetry
- Multiple rounds of conjugation
- Large systerm size.

Possible further improvements: e.g. making connection to dynamical decoupling.

REFERENCES:
[1] Yuval R. Sanders *et al*, NJP (2015)
[2] Charles H. Bennett *et al*, PRA (1996)

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